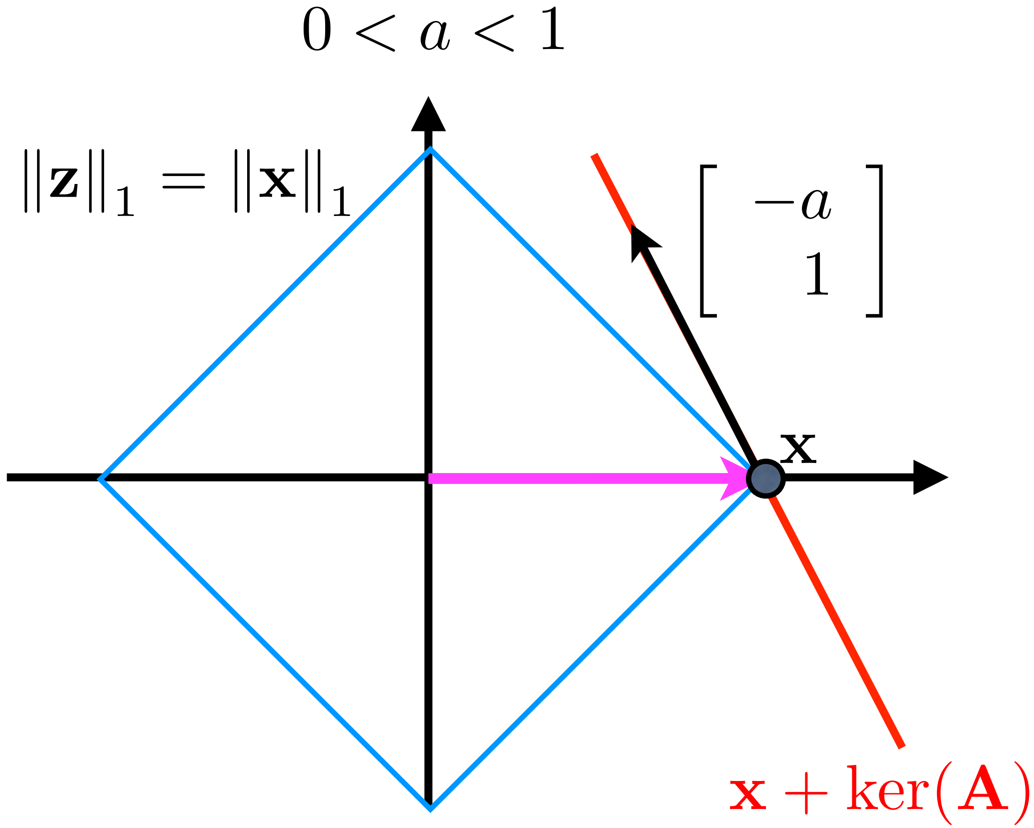
Graphical illustration with *p=1* and **x** sparse



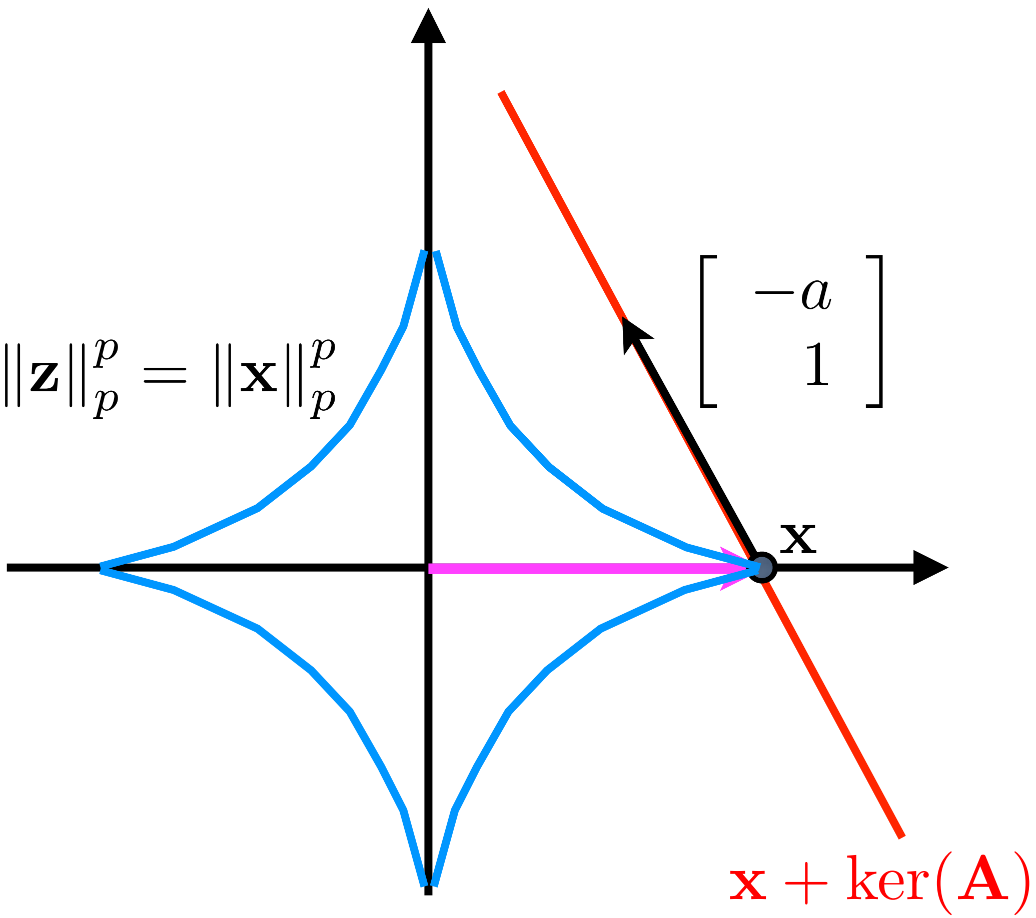
$$\mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in \mathbb{R}^2$$

$$\|\mathbf{x}\|_0 = 1$$

$$\mathbf{A} = \begin{bmatrix} 1 & a \end{bmatrix} \in \mathbb{R}^{1\times 2}$$

$$\ker(\mathbf{A}) \propto \begin{bmatrix} -a \\ 1 \end{bmatrix}$$

Graphical illustration with 0<*p*<1 and **x** sparse



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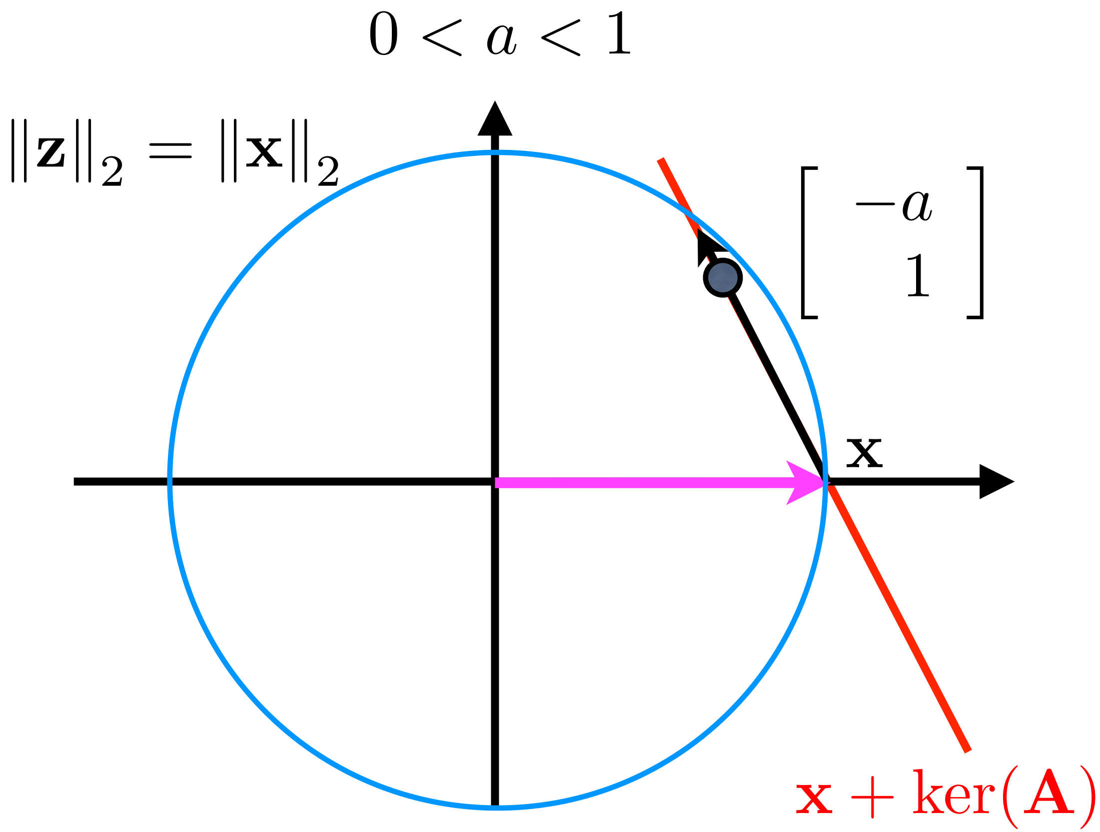
$$\ker(\mathbf{A}) \propto \begin{bmatrix} -a \\ 1 \end{bmatrix}$$

$$0 < a < 1$$

Graphical illustration with p=2 and $\mathbf{x}$ sparse



$$\mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in \mathbb{R}^2$$

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$$\mathbf{A} = \begin{bmatrix} 1 & a \end{bmatrix} \in \mathbb{R}^{1\times 2}$$

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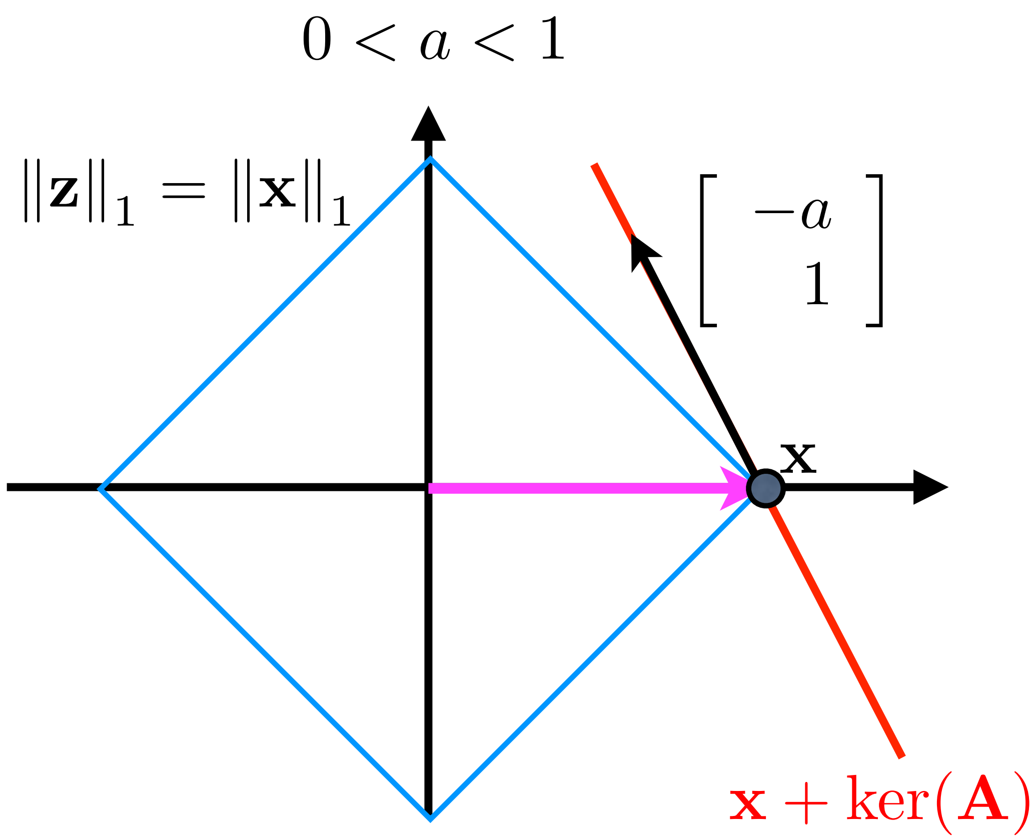
Graphical illustration with *p=1* and **x** sparse



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$$\|\mathbf{x}\|_0 = 1$$

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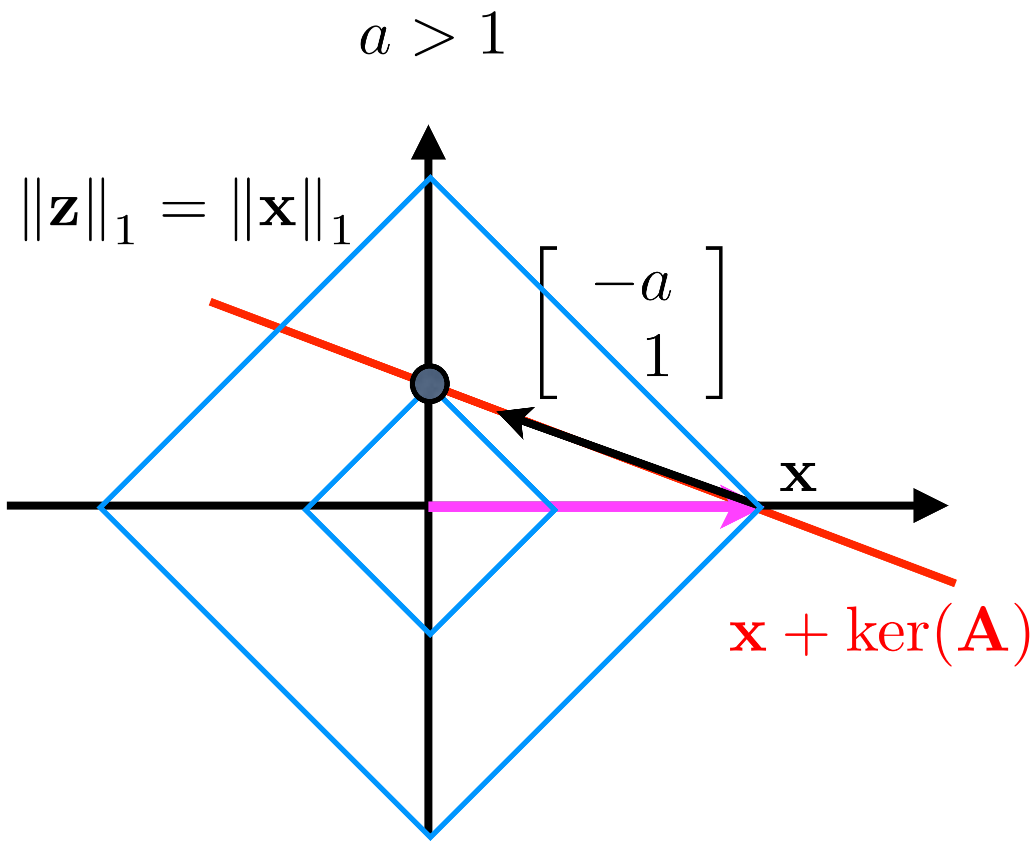
$$\ker(\mathbf{A}) \propto \begin{bmatrix} -a \\ 1 \end{bmatrix}$$

Graphical illustration with *p=1* and **x** sparse



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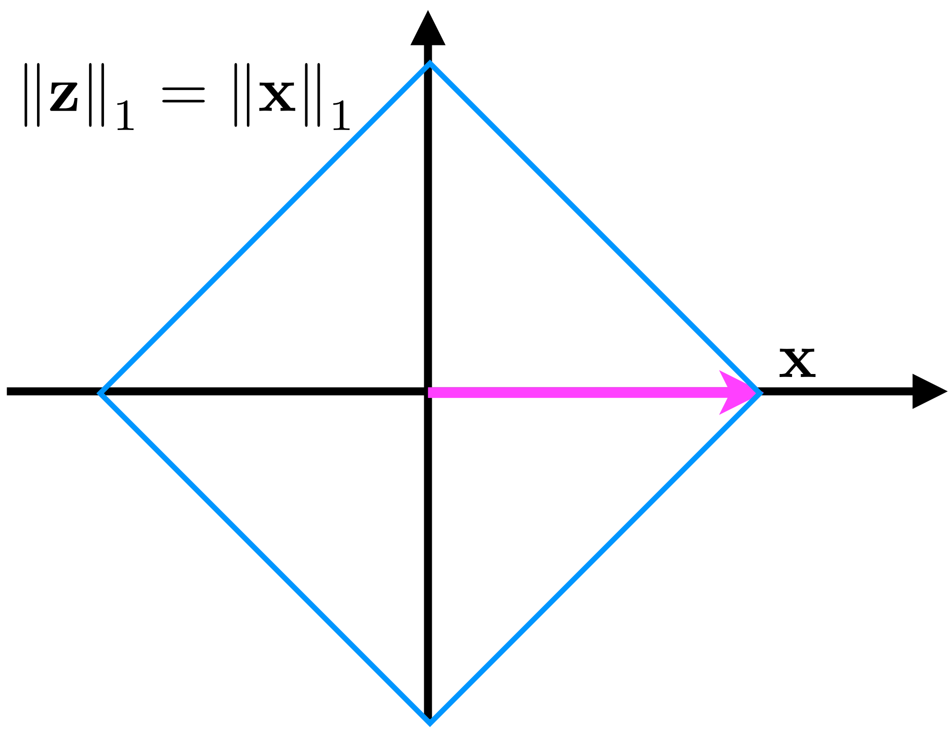
Recovery of **x** clearly depends on ker(**A**)

Graphical illustration with *p=1* and **x** sparse



$$\mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in \mathbb{R}^2$$

$$\|\mathbf{x}\|_0 = 1, S = \{1\}$$

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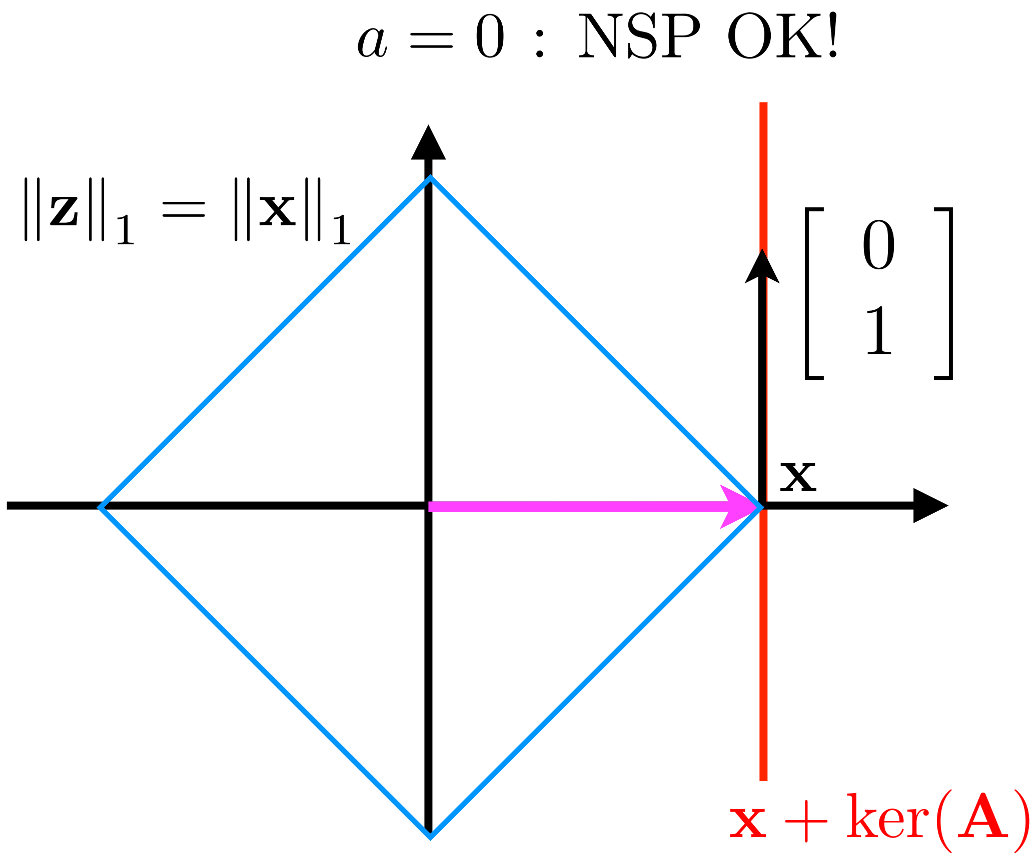
$$(\mathrm{NSP}_p(S)) : \|\mathbf{v}_S\|_1^1 < \|\mathbf{v}_{\bar{S}}\|_1^1 \equiv |a| < 1$$

Graphical illustration with *p=1* and **x** sparse



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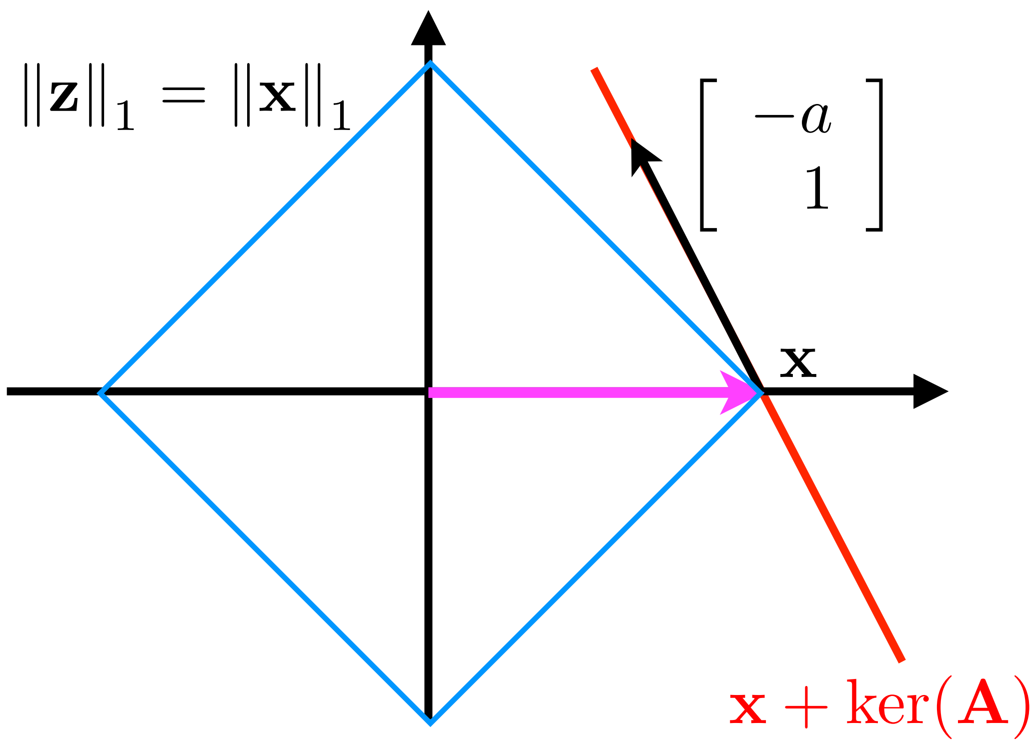
Graphical illustration with *p=1* and **x** sparse

$0 < a < 1$: NSP OK!



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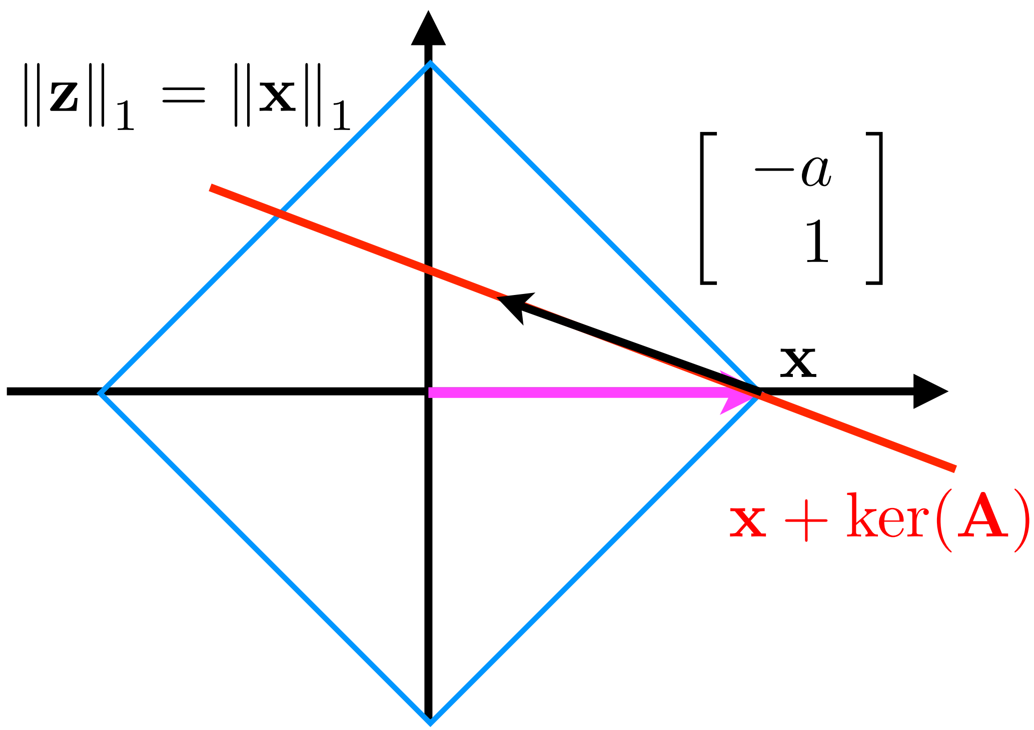
Graphical illustration with *p=1* and **x** sparse

$a > 1:$ failure NSP!



$$\mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in \mathbb{R}^2$$

$$\|\mathbf{x}\|_0 = 1\,, S = \{1\}$$

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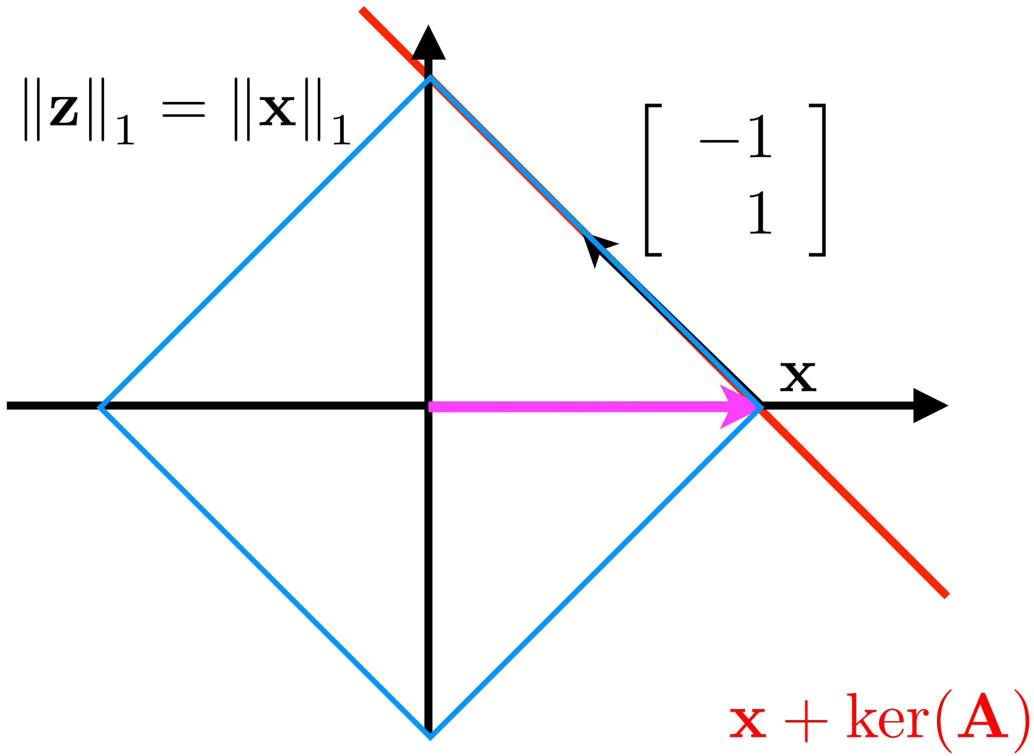
$$(\mathrm{NSP}_p(S)) : \|\mathbf{v}_S\|_1^1 < \|\mathbf{v}_{\bar{S}}\|_1^1 \equiv |a| < 1$$

Graphical illustration with *p=1* and **x** sparse

$a = 1$: failure NSP!



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$$(\mathrm{NSP}_p(S)) : \|\mathbf{v}_S\|_1^1 < \|\mathbf{v}_{\bar{S}}\|_1^1 \equiv |a| < 1$$

Exercise

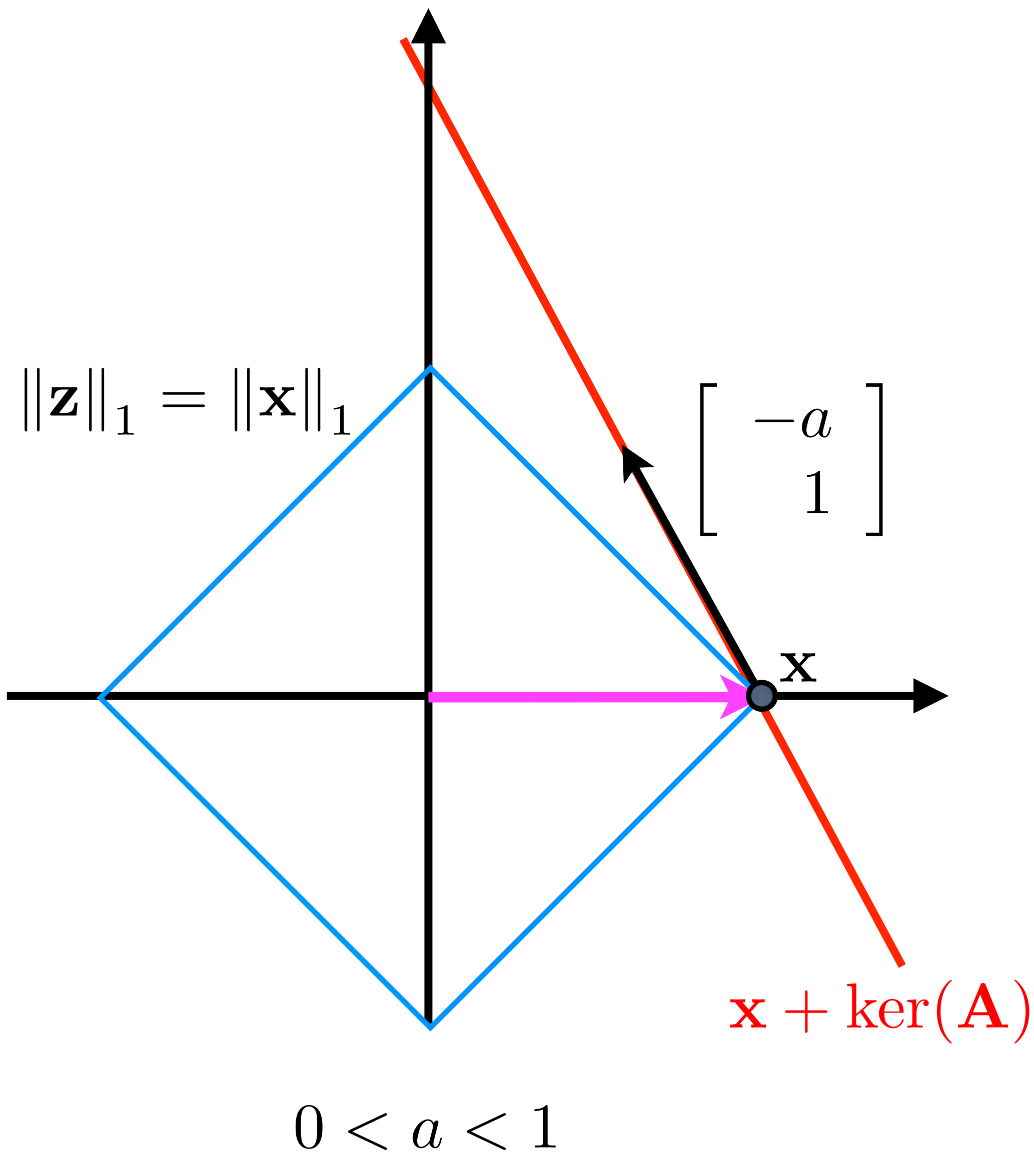
Particularize $\text{NSP}_1(S)$ and $\text{NSP}_0(S)$ to the following dictionary:

$$\mathbf{A} = \begin{bmatrix} 1 & a \end{bmatrix} \in \mathbb{R}^{1\times 2}$$

for $S=\{1\}$.

If $-1<a<1$, can we recover S with $p=1$? and $p=0$?

Graphical illustration with *p=1* and **x** sparse

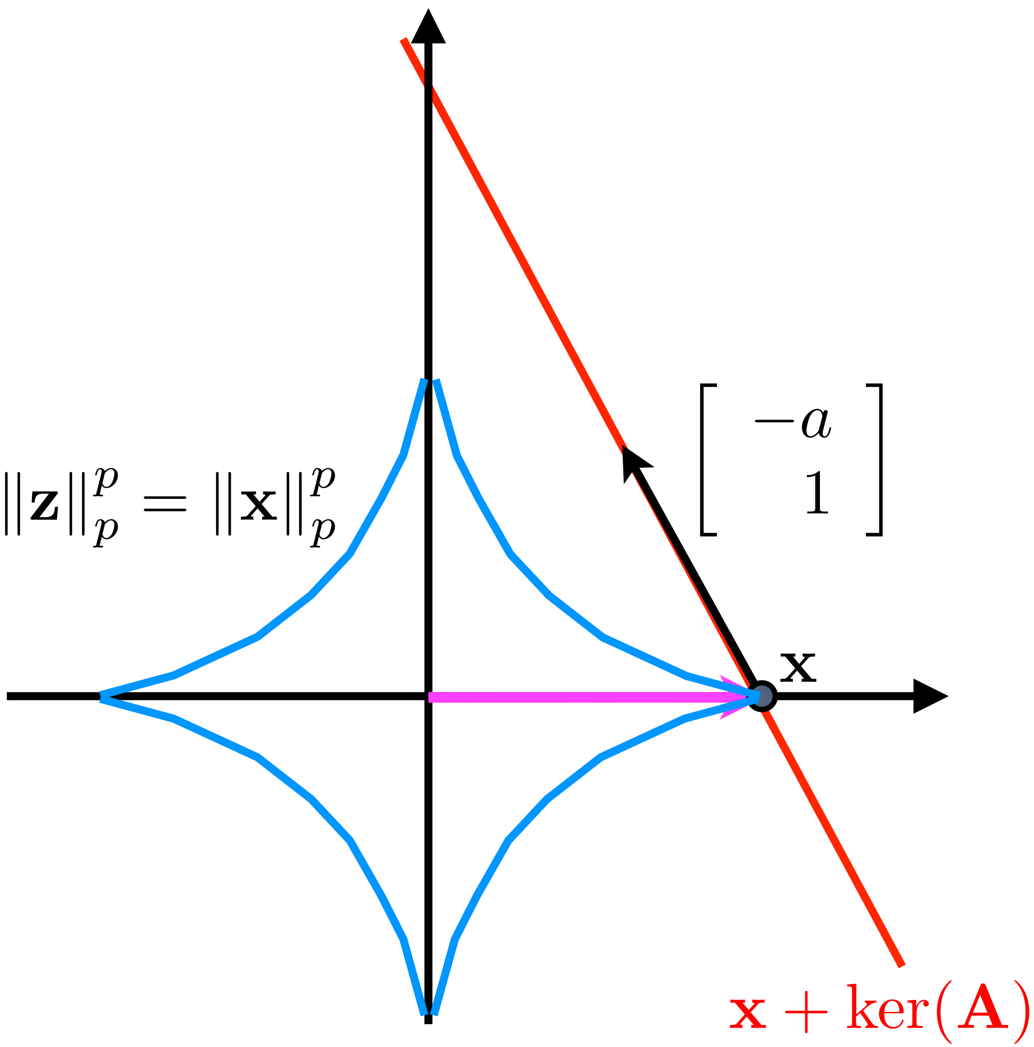


$$\mathrm{NSP}_p(S) : |a|^p < 1^p$$

$$p = 1 : |a| < 1$$

$$\mathrm{NSP}_1(S) \text{ OK!}$$

Graphical illustration with 0<*p*<1 and **x** sparse

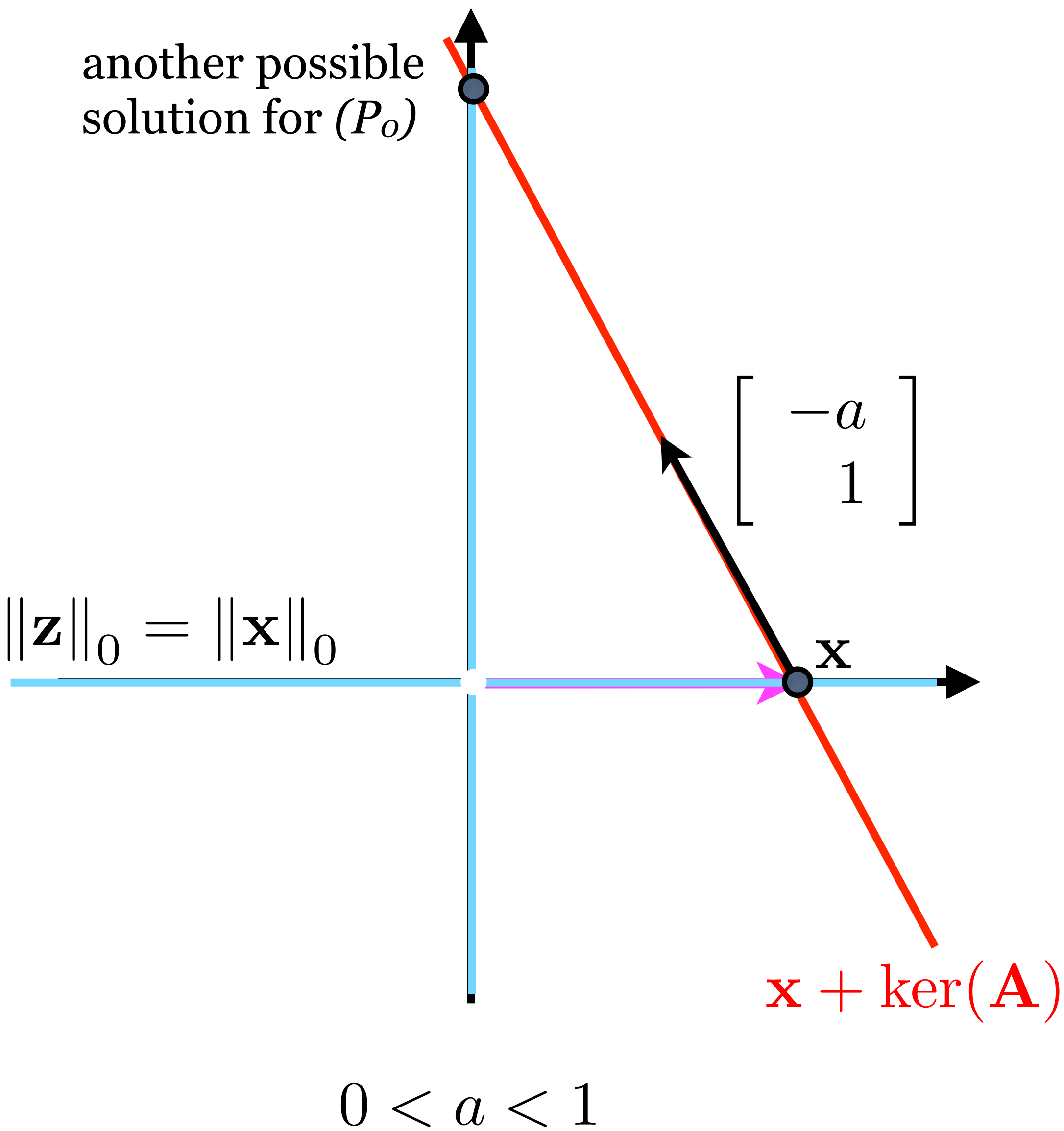


$0 < a < 1$

$\mathrm{NSP}_p(S) : |a|^p < 1^p$

$\mathrm{NSP}_p(S)$ OK!

Graphical illustration with *p=0* and **x** sparse



$$\text{NSP}_p(S) : |a|^p < 1^p$$

$$p = 0 : 1 < 1$$

$$\text{NSP}_0(S) \text{ KO!}$$