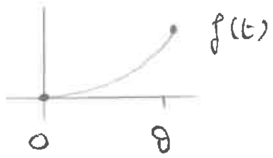
$f(t) = \frac{3t^2}{\theta^3} \mathbb{1}_{[0,\theta]}(t)$



1.11.

1°. $F(x) = \mathbb{P}[X \leq x]$.

$$= \int_0^x \frac{3t^2}{\theta^3} dt = \frac{t^3}{\theta^3}\Big]_0^x = \left(\frac{x}{\theta}\right)^3 \quad \text{si } x \in]0,\theta[$$

0 si $x < 0$

1 si $x > \theta$

$$\mathbb{E}[X] = \frac{3}{\theta^3}\int_0^\theta t^3 dt = \frac{3\theta}{4}$$

$$\mathbb{E}[X^2] = \frac{3}{\theta^3} \times \frac{1}{5} t^5\Big]_0^\theta = \frac{3}{5}\theta^2 \Rightarrow \mathbb{V}(X) = \frac{3}{5}\theta^2 - \frac{9}{16}\theta^2 = \frac{3}{80}\theta^2$$

2°. $\mathbb{E}[S_n] = \mathbb{E}[X] = \frac{3}{4}\theta \Rightarrow \hat{S}_n = \frac{4}{3} S_n$

$$\operatorname{Var} \hat{S}_n = \frac{1}{n} \times \frac{16}{9} \operatorname{Var}(X_1) = \frac{1}{n} \times \frac{16}{9} \times \frac{3}{80}\theta^2 = \frac{\theta^2}{15n} \underset{n\to\infty}{\longrightarrow} 0$$

donc $\mathbb{E}[(\hat{S}_n - \theta)^2] \underset{n\to\infty}{\longrightarrow} 0$.

3°. $F_T(x) = F(x)^n = \left(\frac{x}{\theta}\right)^{3n} \Rightarrow f_T(x) = 3n\left(\frac{x}{\theta}\right)^{3n-1} \mathbb{1}_{[0,\theta]}(x)$

↳ c'est la loi de $X_{(n)}$ stat d'ordre

$$\mathbb{E}[T_n] = \frac{3n}{\theta^{3n-1}} \frac{1}{3n+1} x^{3n+1}\Big]_0^\theta = \theta \times \frac{3n}{3n+1}$$

$$\mathbb{E}[T_n^2] = \frac{3n}{\theta^{3n-1}} \frac{1}{3n+2} \theta^{3n+2} = \frac{3n}{3n+2}\theta^2$$

$$\Rightarrow \operatorname{Var} T = \| T_n - \theta \|_2^2 = \theta^2\left[\frac{3n}{3n+2} - \left(\frac{3n}{3n+1}\right)^2\right] = \frac{\theta^2}{(\cdot)(\cdot)}[27n^3 + 18n^2 + 3n - 27n^3 - 18n^2]$$

$$= \frac{3n\,\theta^2}{(3n+2)(3n+1)^2} \longrightarrow 0 \quad n\to\infty \qquad T_n \xrightarrow{L^2} \theta$$

4°. $$\mathbb{E}[X^{1/n}] = \int_0^\theta x^{1/n} \frac{3x^2}{\theta^3} dx = \frac{3}{\theta^3} \times \frac{1}{3+\frac{1}{n}} x^{3+1/n}\Big]_0^\theta = \frac{3n}{3n+1}\theta^{1/n}$$

$\mathbb{E}[Z_n] = \mathbb{E}[X^{1/n}]^n$ par ⊥⊥ des X_i

$= \left(\frac{3n}{3n+1}\right)^n \theta \qquad \lim_{n\to\infty}\left(\frac{3n}{3n+1}\right)^n = e^{-1/3} \qquad$ donc si $a = e^{1/3}$

$\hat{Z}_n = a Z_n$ est non biaisé

$$\mathbb{E}[Z_n^2] = \mathbb{E}[X^{2/n}]^n = \left(\frac{3n}{3n+2}\theta^{2/n}\right)^n$$

$$\operatorname{Var} Z_n = \left(\frac{3n}{3n+2}\right)^n \theta^2 - \left(\frac{3n}{3n+1}\right)^{2n}\theta^2 = \theta^2 \times \frac{2n}{(3n+2)(3n+1)} + o\left(\frac{1}{n}\right)$$

$\longrightarrow 0 \quad n \to \infty \qquad Z_n$ et $\hat{Z}_n$ cv vers θ ds L^2

5°. $r_n(\hat{S}_n) \overset{\infty}{\sim} \frac{\theta^2}{15n} \qquad r_n(\hat{Z}_n) \overset{\infty}{\sim} \frac{2}{9}\frac{\theta^2}{n} \Rightarrow \hat{S}_n$ décroît plus vite.